# Advances in UQ Algorithms for Wind Energy Applications

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# Research in Scalable UQ Methods

For production UQ analyses, we prefer fast converging global methods:

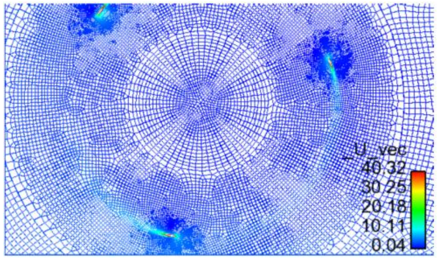
- Local approximate methods (reliability methods, moment-based methods) exhibit significant errors in presence of multimodal/nonsmooth/highly nonlinear responses
- MC/LHS are robust with dimension-independent conv., but rates can be unacceptably slow

Spectral methods (e.g., PCE) provide a more effective balance of robustness and efficiency, especially when solution smoothness can be exploited

- Exponential growth in expansion cardinality with *n* and *p*
- collocation requirements are on the order of the number of terms


To mitigate the curse of dimensionality:

- *A priori* model reduction methods (e.g., POD, Karhunen-Loeve)
- Goal-oriented adaptive refinement to reduce effective dimension
- Adjoint techniques [given *n* (random dimension) > *m* (response QoI)]
- Sparsity detection methods: compressive sensing, least interpolation




Build on this foundation:

- Error balancing framework
- Multiple model forms
  - Model hierarchy → multifidelity UQ (LF: EOLO, FAST, CACTUS; HF: URANS, DG LES)
  - Epistemic model form → mixed aleatory-epistemic/continuous-discrete UQ

# Non-Intrusive Stochastic Expansions: Polynomial Chaos and Stochastic Collocation

***Polynomial chaos:*** spectral projection using orthogonal polynomial basis fns

$$R = \sum_{j=0}^{P} \alpha_j \Psi_j(\boldsymbol{\xi})$$

using

$$\begin{aligned}
\Psi_0(\xi) &= \psi_0(\xi_1)\,\psi_0(\xi_2) = 1 \\
\Psi_1(\xi) &= \psi_1(\xi_1)\,\psi_0(\xi_2) = \xi_1 \\
\Psi_2(\xi) &= \psi_0(\xi_1)\,\psi_1(\xi_2) = \xi_2 \\
\Psi_3(\xi) &= \psi_2(\xi_1)\,\psi_0(\xi_2) = \xi_1^2 - 1 \\
\Psi_4(\xi) &= \psi_1(\xi_1)\,\psi_1(\xi_2) = \xi_1\xi_2 \\
\Psi_5(\xi) &= \psi_0(\xi_1)\,\psi_2(\xi_2) = \xi_2^2 - 1
\end{aligned}$$

| Distribution | Density function | Polynomial | Weight function | Support range |
|---|---|---|---|---|
| Normal | $\frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}$ | Hermite $He_n(x)$ | $e^{\frac{-x^2}{2}}$ | $[-\infty, \infty]$ |
| Uniform | $\frac{1}{2}$ | Legendre $P_n(x)$ | 1 | $[-1, 1]$ |
| Beta | $\frac{(1-x)^\alpha (1+x)^\beta}{2^{\alpha+\beta+1} B(\alpha+1, \beta+1)}$ | Jacobi $P_n^{(\alpha,\beta)}(x)$ | $(1-x)^\alpha (1+x)^\beta$ | $[-1, 1]$ |
| Exponential | $e^{-x}$ | Laguerre $L_n(x)$ | $e^{-x}$ | $[0, \infty]$ |
| Gamma | $\frac{x^\alpha e^{-x}}{\Gamma(\alpha+1)}$ | Generalized Laguerre $L_n^{(\alpha)}(x)$ | $x^\alpha e^{-x}$ | $[0, \infty]$ |

- Estimate $\alpha_j$ using regression or numerical integration: sampling, tensor quadrature, sparse grids, or cubature

$$\alpha_j = \frac{\langle R, \Psi_j \rangle}{\langle \Psi_j^2 \rangle} = \frac{1}{\langle \Psi_j^2 \rangle} \int_\Omega R\, \Psi_j\, \varrho(\boldsymbol{\xi})\, d\boldsymbol{\xi}$$

$$\langle \Psi_j^2 \rangle = \prod_{i=1}^{n} \langle \psi_{m_i^j}^2 \rangle$$

***Stochastic collocation:*** instead of estimating coefficients for known basis functions, form interpolants for known coefficients

- ***Global:*** Lagrange (values) or Hermite (values+derivatives)
- ***Local:*** linear (values) or cubic (values+gradients) splines

$$R = \sum_{j=1}^{N_p} r_j \boldsymbol{L}_j(\boldsymbol{\xi})$$

$$L_i = \prod_{\substack{j=1 \\ j \neq i}}^{m} \frac{x - x_j}{x_i - x_j} \Longrightarrow R(\boldsymbol{\xi}) \cong \sum_{j_1=1}^{m_{i_1}} \cdots \sum_{j_n=1}^{m_{i_n}} r\left(\xi_{j_1}^{i_1}, \ldots, \xi_{j_n}^{i_n}\right) \left(L_{j_1}^{i_1} \otimes \cdots \otimes L_{j_n}^{i_n}\right)$$

Sparse interpolants formed using $\Sigma$ of tensor interpolants

- Tailor expansion form:
  - p-refinement: anisotropic tensor/sparse, generalized sparse
  - h-refinement: local bases with dimension & local refinement
- Method selection: fault tolerance, decay, sparsity, error est.

# Non-Intrusive Stochastic Expansions: Polynomial Chaos and Stochastic Collocation

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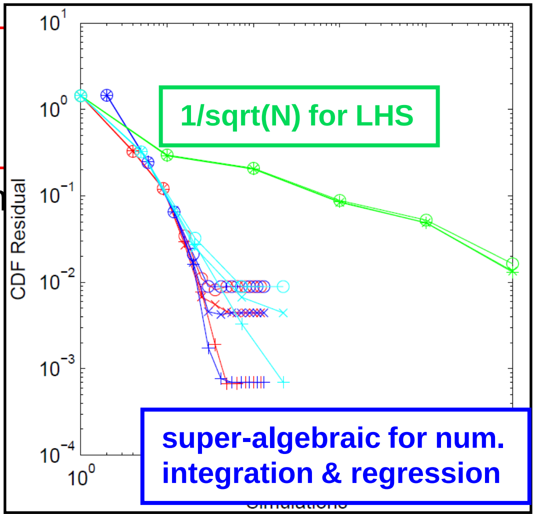
Sparse interpolants formed using $\Sigma$ of tensor in




- Tailor expansion form:
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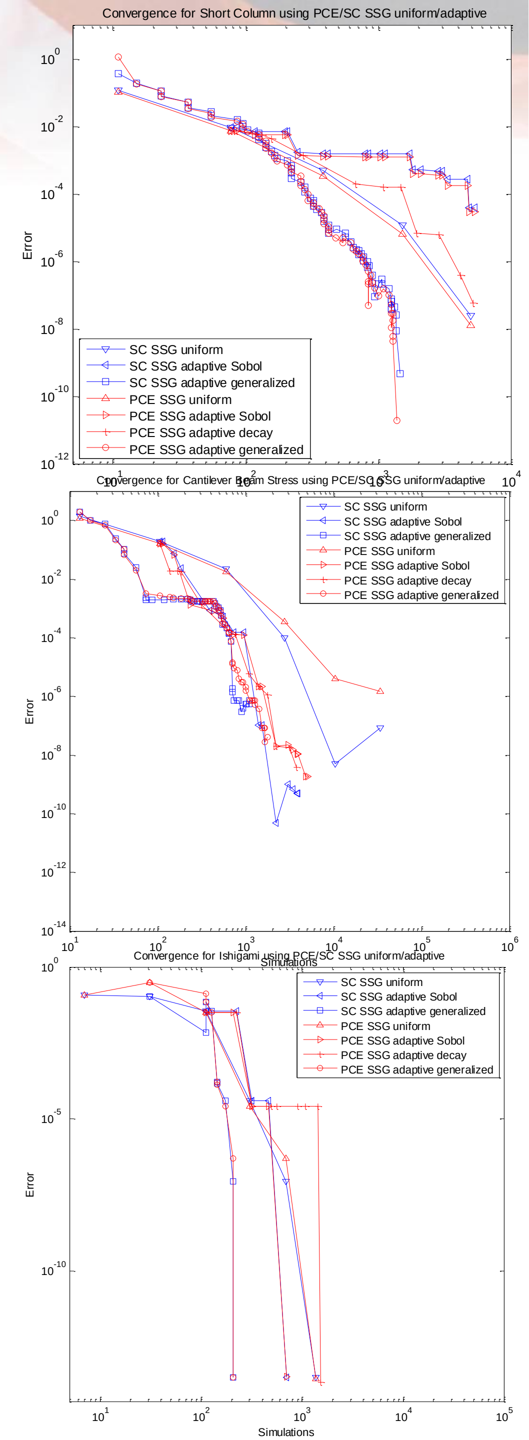
# Stochastic Expansions on Structured Grids:
# Adaptive Collocation Methods

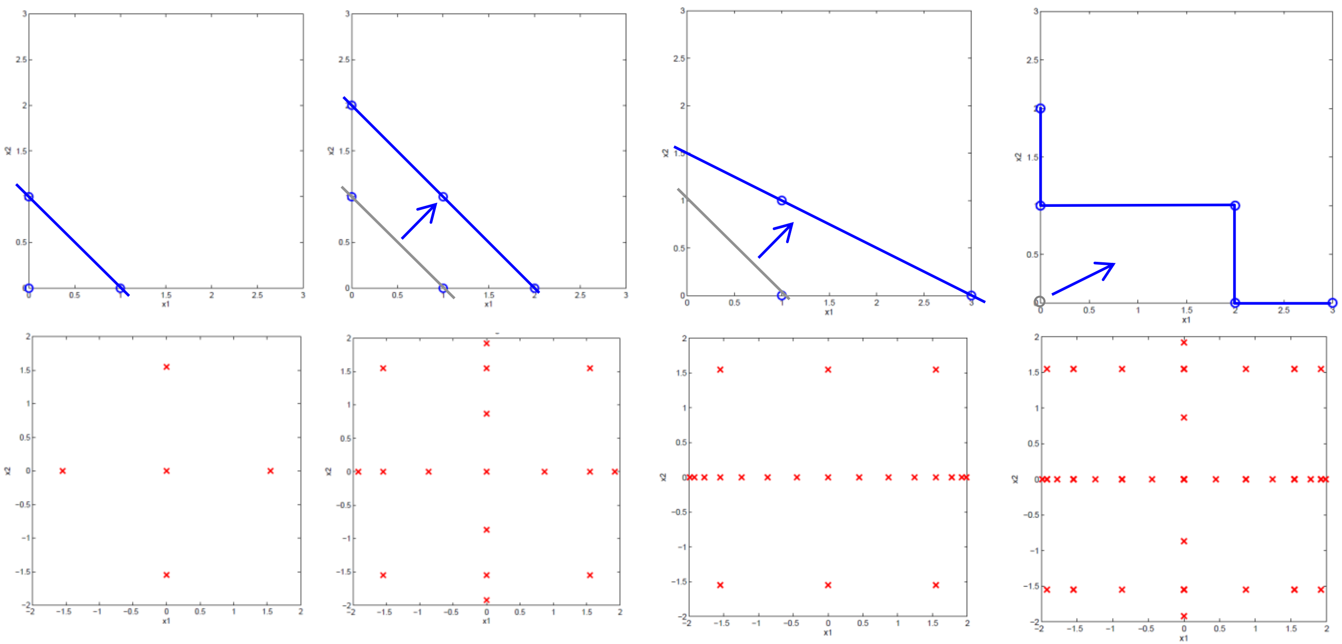
## *Polynomial order (p-) refinement approaches:*

- **Uniform:** *isotropic* tensor/sparse grids
  - *Increment grid:* increase order/level, ensure change (restricted/nested)
  - *Assess convergence:* $L^2$ change in response covariance
- **Adaptive:** *anisotropic* tensor/sparse grids

$$w\underline{\alpha} - |\underline{\alpha}| < \sum_{n=1}^{d} (i_n - 1)\alpha_n \le w\underline{\alpha}$$

  - **PCE/SC:** variance-based decomp. → total Sobol' indices → anisotropy
  - **PCE:** spectral coefficient decay rates → anisotropy
- **Goal-oriented adaptive:** *generalized* sparse grids
  - **PCE/SC:** change in QOI induced by trial index sets on active front
  - Fine-grained control: frontier not limited by index set constraint



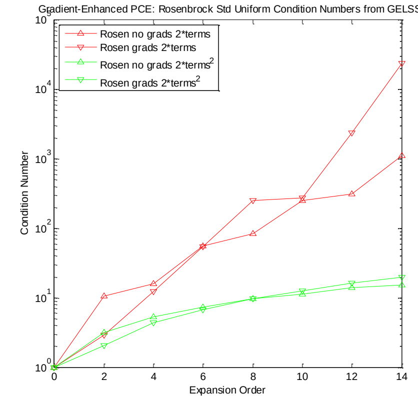



# Extend Scalability: (Adjoint) Derivative-Enhancement

## PCE:

- Linear regression including derivatives
  - Gradients/Hessians → addtnl. eqns.
  - Over-determined: SVD, eq-constrained LS
  - Under-determined: compressive sensing

$$\begin{bmatrix} \vdots & \vdots & & \vdots \\ \pi_{0,j}(\vec{\xi}_i) & \pi_{1,j}(\vec{\xi}_i) & \cdots & \pi_{P,j}(\vec{\xi}_i) \\ \frac{\partial \pi_{0,j}}{\partial \xi_1}(\vec{\xi}_i) & \frac{\partial \pi_{1,j}}{\partial \xi_1}(\vec{\xi}_i) & \cdots & \frac{\partial \pi_{P,j}}{\partial \xi_1}(\vec{\xi}_i) \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial \pi_{0,j}}{\partial \xi_{n_\xi}}(\vec{\xi}_i) & \frac{\partial \pi_{1,j}}{\partial \xi_{n_\xi}}(\vec{\xi}_i) & \cdots & \frac{\partial \pi_{P,j}}{\partial \xi_{n_\xi}}(\vec{\xi}_i) \\ \vdots & \vdots & & \vdots \end{bmatrix} \begin{pmatrix} \vdots \\ \vec{u}^{(m,j)} \\ \vec{u}^{(m+1,j)} \\ \vdots \\ \vec{u}^{(m+n_\xi,j)} \\ \vdots \end{pmatrix} = \begin{pmatrix} \vdots \\ \vec{u}_i \\ \frac{\partial \vec{u}_i}{\partial \xi_1} \\ \vdots \\ \frac{\partial \vec{u}_i}{\partial \xi_{n_\xi}} \\ \vdots \end{pmatrix}$$

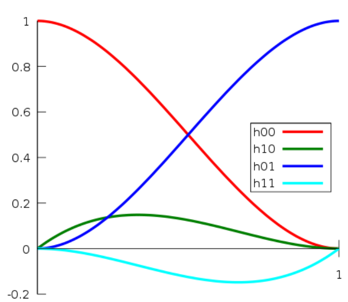



## SC:

- Gradient-enhanced interpolants
  - Local: cubic Hermite splines
  - Global: Hermite interpolating polynomials

$$f = \sum_{i=1}^{N} f_i H_i^{(1)}(x_1) H_i^{(1)}(x_2) H_i^{(1)}(x_3) + \sum_{i=1}^{N} \frac{df_i}{dx_1} H_i^{(2)}(x_1) H_i^{(1)}(x_2) H_i^{(1)}(x_3) + \sum_{i=1}^{N} \frac{df_i}{dx_2} H_i^{(1)}(x_1) H_i^{(2)}(x_2) H_i^{(1)}(x_3) + \sum_{i=1}^{N} \frac{df_i}{dx_3} H_i^{(1)}(x_1) H_i^{(1)}(x_2) H_i^{(2)}(x_3)$$

*Cubic shape fns: type 1 (value) & type 2 (gradient)*

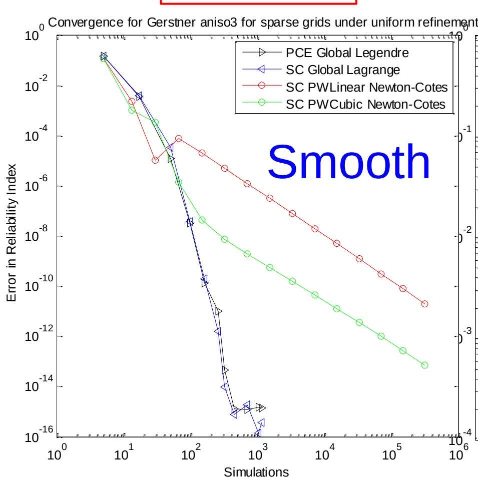


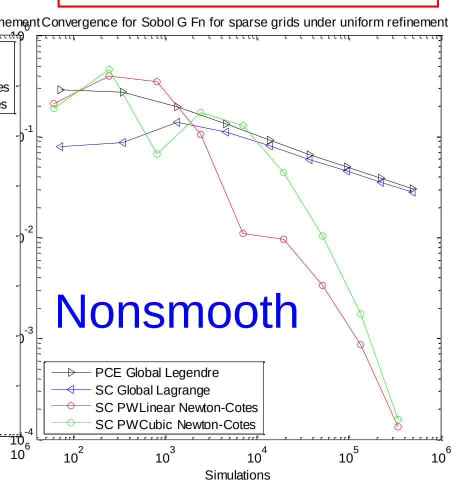

$$\mu = \sum_{i=1}^{N} f_i w_i^{(1)} w_i^{(1)} w_i^{(1)} + \sum_{i=1}^{N} \frac{df_i}{dx_1} w_i^{(2)} w_i^{(1)} w_i^{(1)} + \sum_{i=1}^{N} \frac{df_i}{dx_2} w_i^{(1)} w_i^{(2)} w_i^{(1)} + \sum_{i=1}^{N} \frac{df_i}{dx_3} w_i^{(1)} w_i^{(1)} w_i^{(2)}$$

and similar for higher-order moments

$$e^{-10x^2 - 5y^2}$$

$$f(\mathbf{x}) = 2 \prod_{j=1}^{5} \frac{|4x_j - 2| + a_j}{1 + a_j}; \quad a = [0,1,2,4,8]$$



## Stochastic Expansions on Unstructured Grids:

# Compressive Sensing

$$\begin{bmatrix} f(\mathbf{x}^{(1)}) \\ f(\mathbf{x}^{(2)}) \\ \vdots \\ f(\mathbf{x}^{(N)}) \end{bmatrix} = \begin{bmatrix} 1 & \Phi_2(\mathbf{x}^{(1)}) & \Phi_2(\mathbf{x}^{(1)}) & \dots & \Phi_P(\mathbf{x}^{(1)}) \\ 1 & \Phi_1(\mathbf{x}^{(2)}) & \Phi_2(\mathbf{x}^{(2)}) & \dots & \Phi_P(\mathbf{x}^{(2)}) \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & \Phi_1(\mathbf{x}^{(N)}) & \Phi_2(\mathbf{x}^{(N)}) & \dots & \Phi_P(\mathbf{x}^{(N)}) \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ \vdots \\ c_P \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_N \end{bmatrix}$$

or in matrix notation

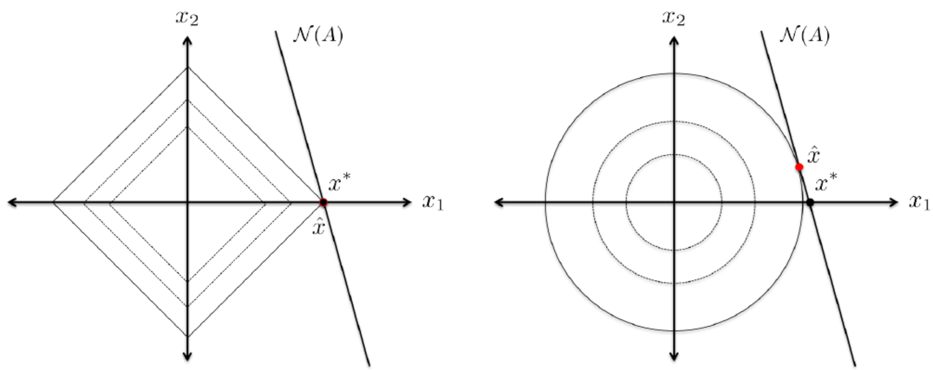
$$\mathbf{b} = \mathbf{A}\mathbf{x} + \varepsilon$$

and find the minimum norm solution

$$\min_{\mathbf{x}} \quad \|\mathbf{A}\mathbf{x} - \mathbf{b}\|_2$$

or ( more recently ) find a sparse solution

$$\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{such that} \quad \|\mathbf{A}\mathbf{x} - \mathbf{b}\|_2 \leq \varepsilon$$




(a) CS methodology ($\ell_1$ objective) (b) Pseudo-inverse ($\ell_2$ objective)

BP

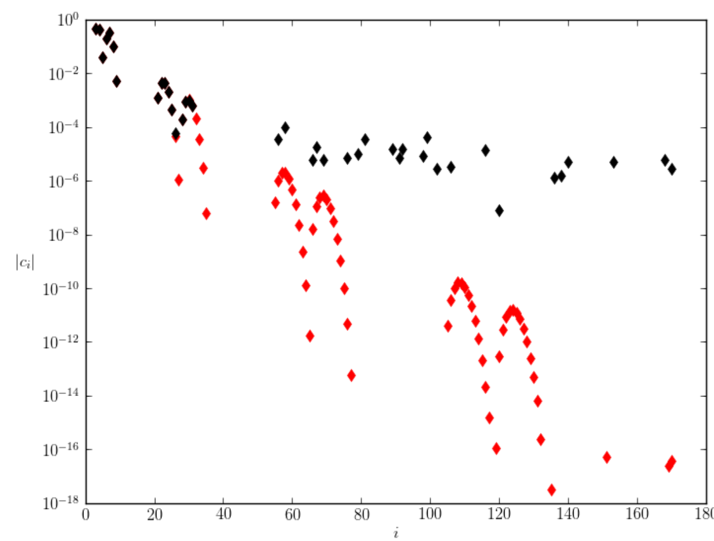
$$\mathbf{c} = \arg\min \|\mathbf{c}\|_{\ell^1} \quad \text{such that} \quad \mathbf{\Phi c} = \mathbf{y}$$

BPDN and OMP

$$\mathbf{c} = \arg\min \|\mathbf{c}\|_{\ell^1} \quad \text{such that} \quad \|\mathbf{\Phi c} - \mathbf{y}\|_{\ell^2} \leq \varepsilon$$

LASSO and LARS

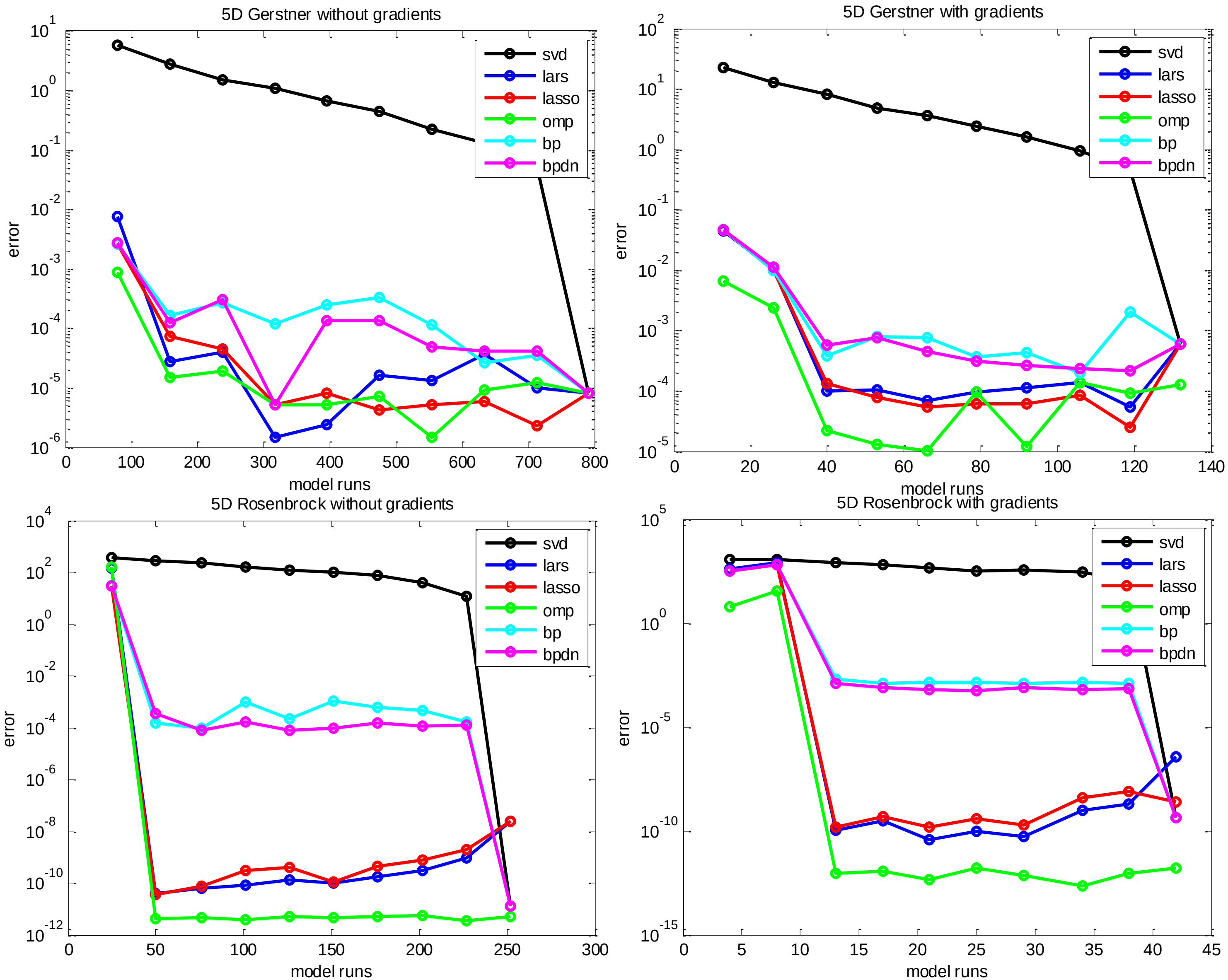
$$\mathbf{c} = \arg\min \|\mathbf{\Phi c} - \mathbf{y}\|^2_{\ell^2} \quad \text{such that} \|\mathbf{x}\|_{\ell^1} \leq \tau$$



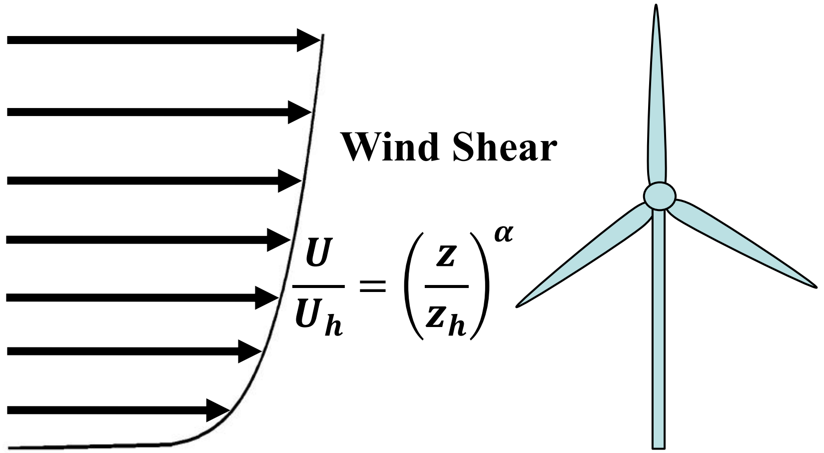
# Comparison of CS Methods with SVD for under-determined PCE

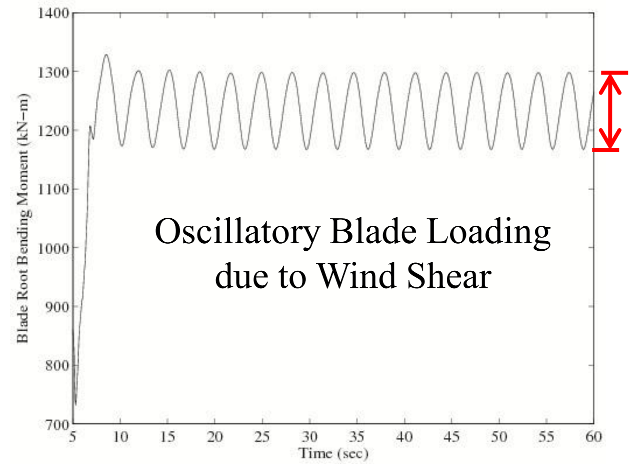
**5D Gerstner without/with gradients**

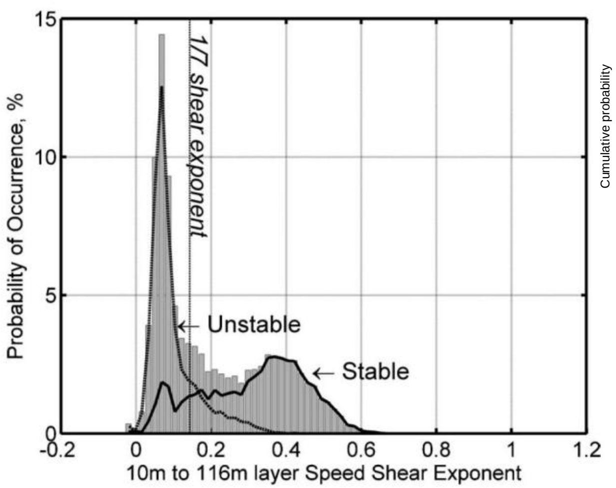
**5D Rosenbrock without/with gradients**

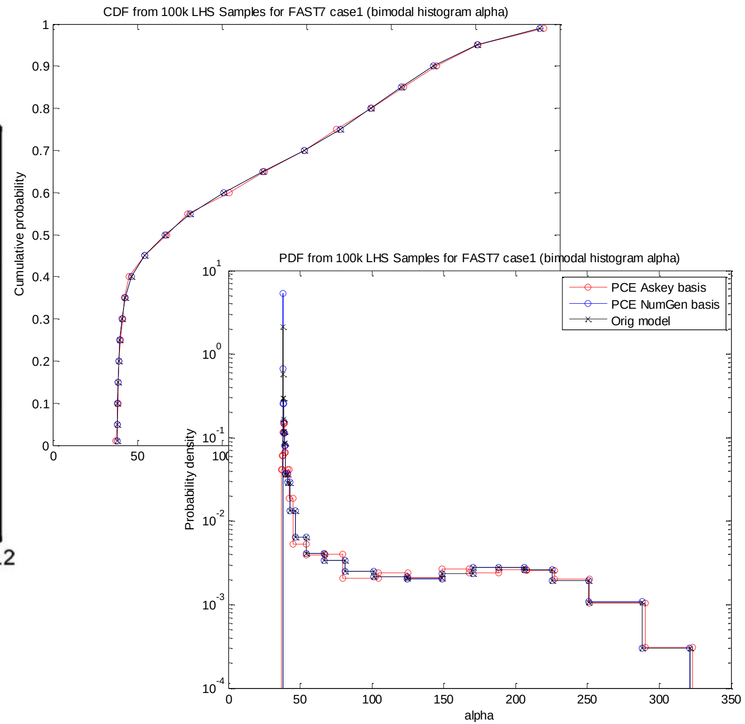


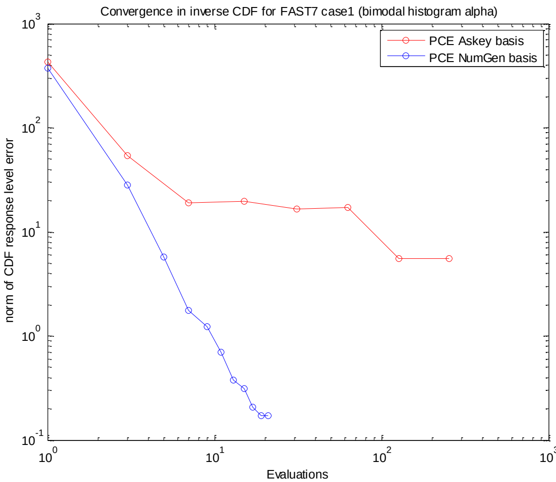
# FAST: HAWT Loads due to Uncertain Wind Shear

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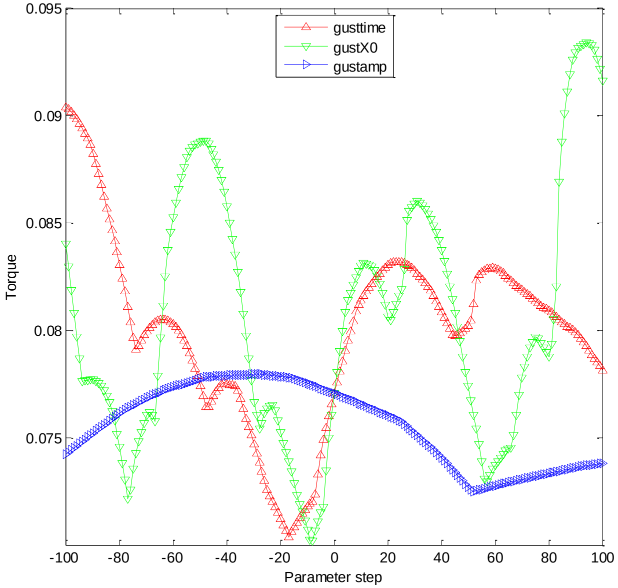
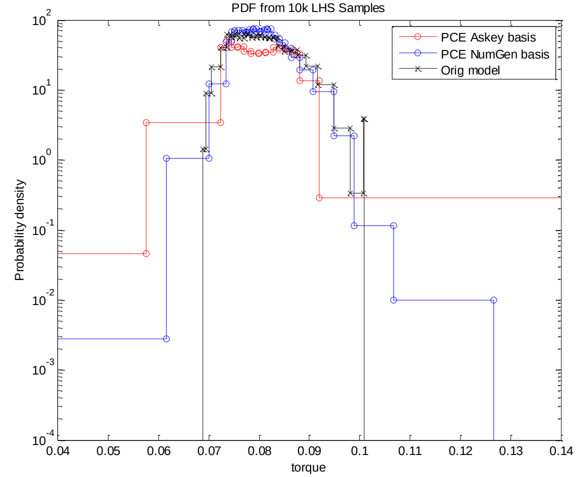
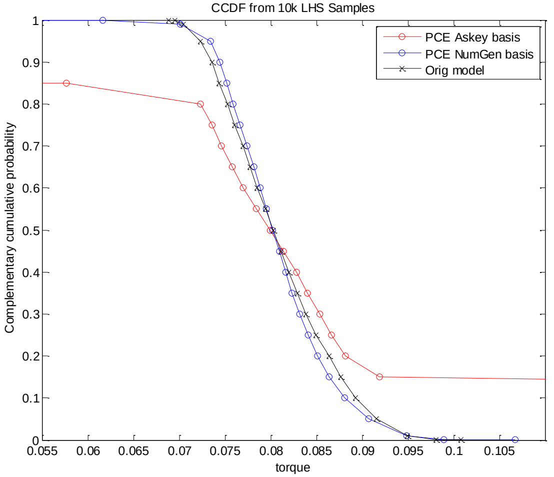
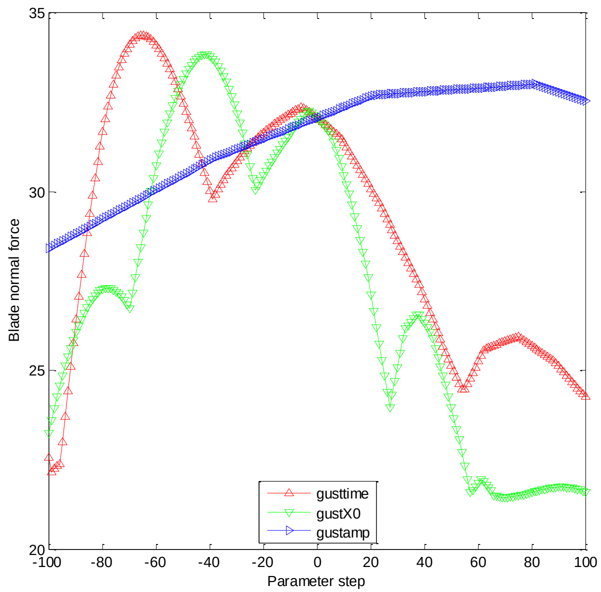
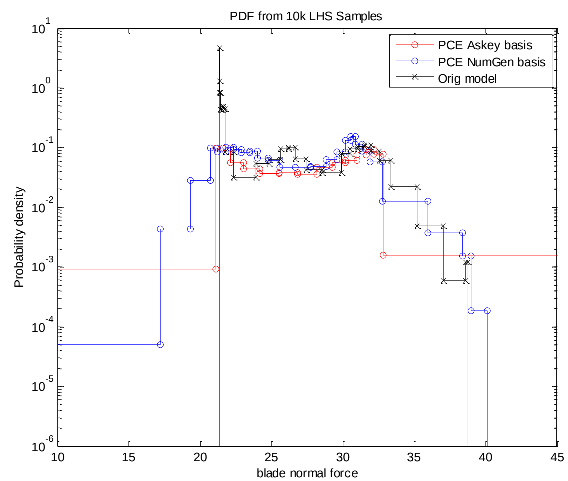
# CACTUS: VAWT with Uncertain Gust Loading (V1)

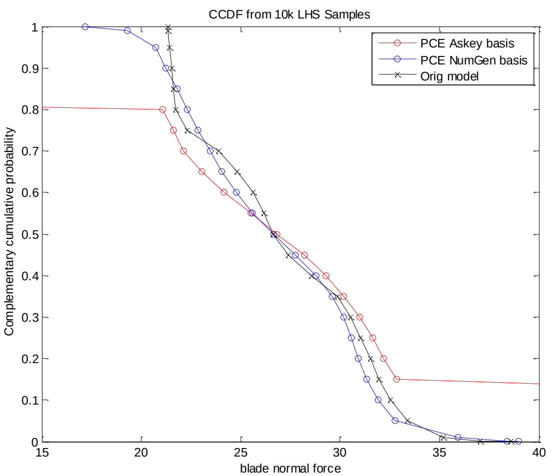
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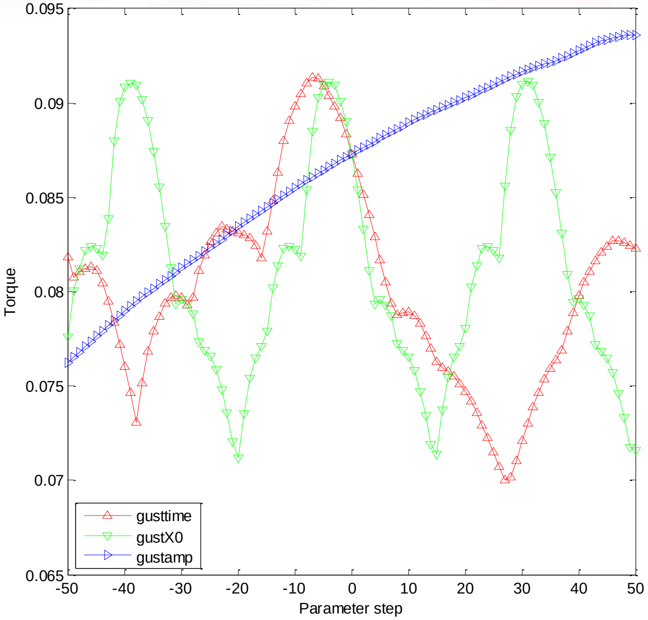
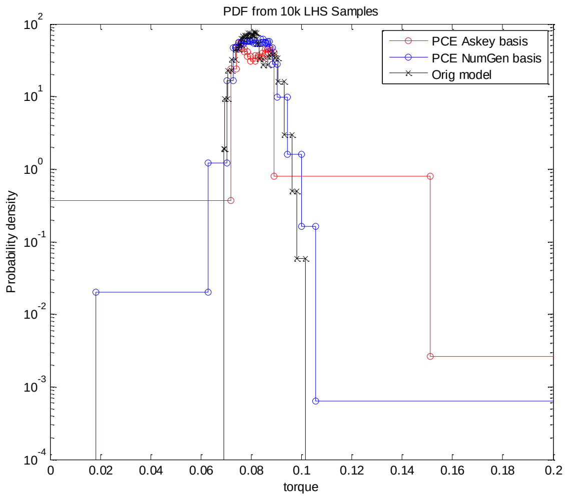
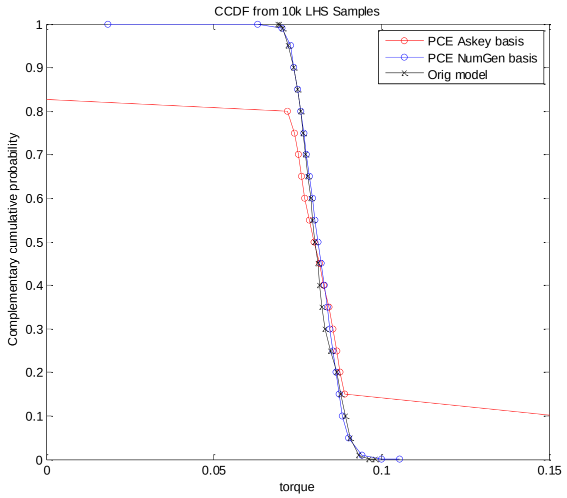
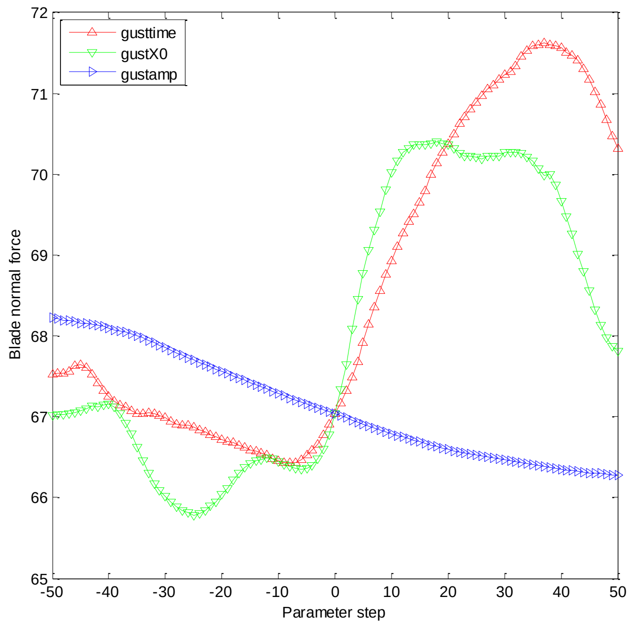
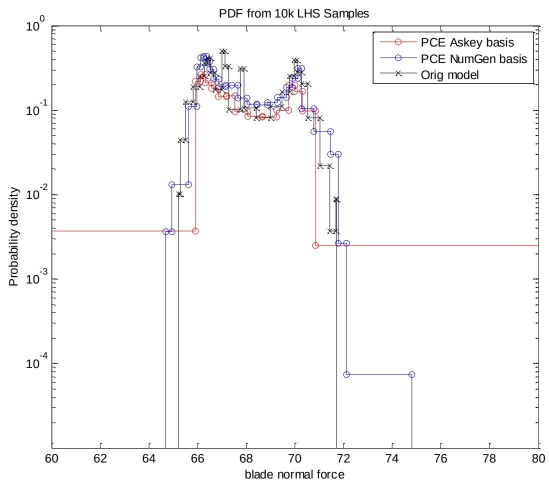
# CACTUS: VAWT Loads due to Uncertain Gust (V2)

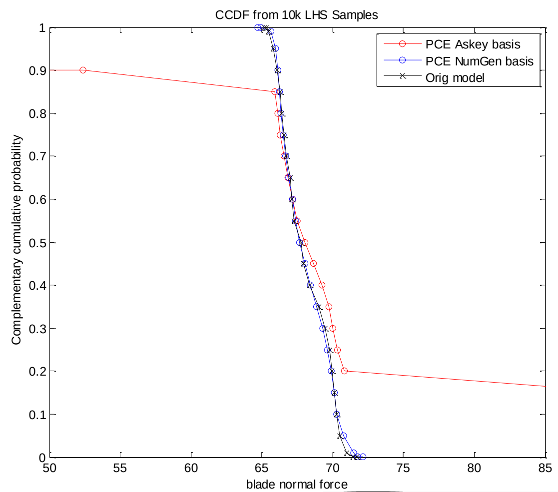
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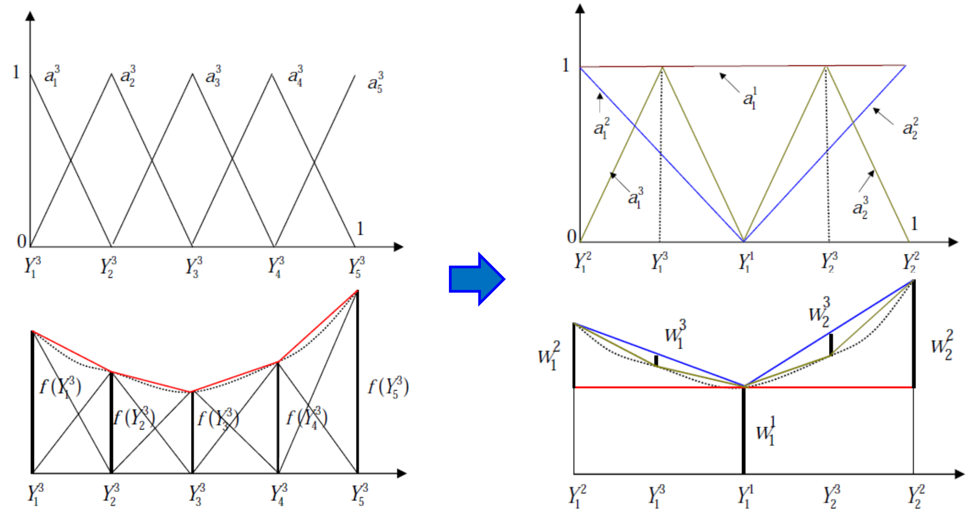
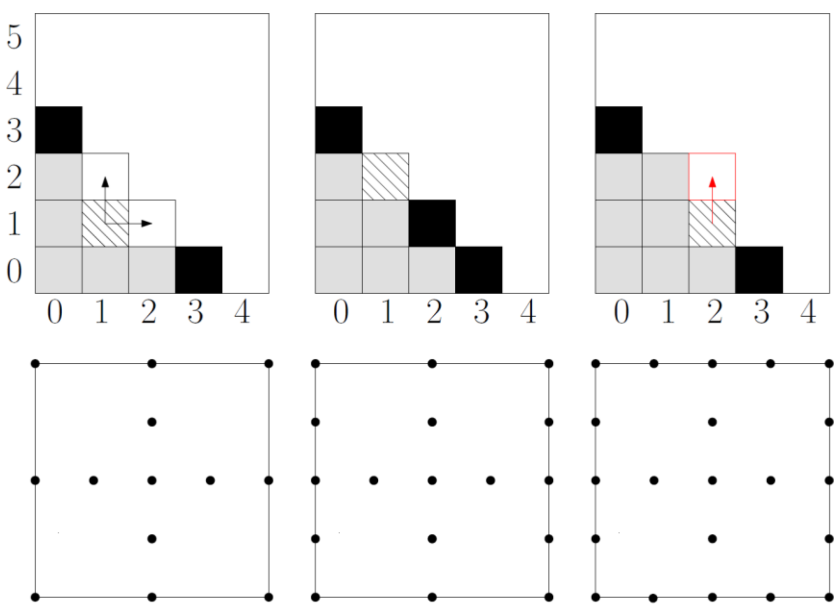
# Local Error Estimation with Hierarchical Value/Gradient Surpluses

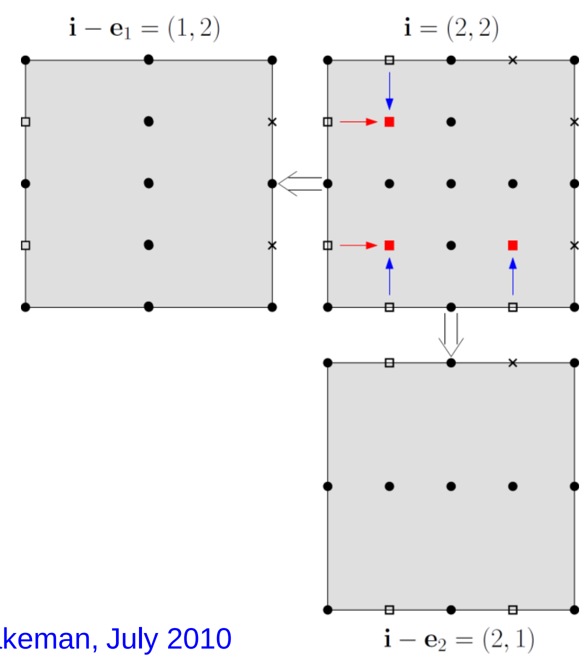
Hierarchical basis:

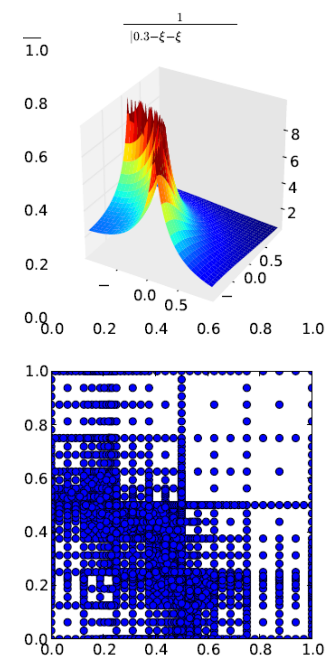
- *Improved precision* in QoI increments
- Surpluses provide error estimates for *local refinement* using local/global hierarchical interpolants
- New error indicators under development that leverage both *value and gradient surpluses*



Hierarchical linear splines; from X. Ma, Ph.D. dissertation, Cornell Univ., 2010





From J. Jakeman, July 2010

# Error budgets

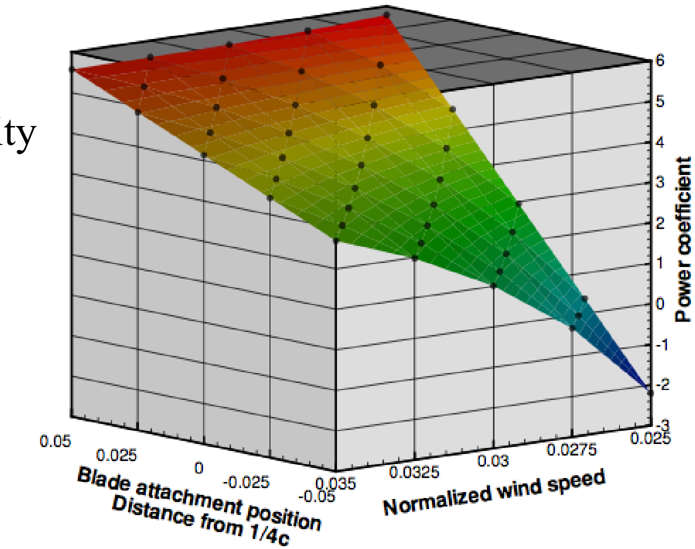
Two uncertain parameters:
Blade attachment point, freestream velocity
Objective function: Mean power produced

Given an accuracy requirement (e.g. confidence in power prediction), need to budget sources of errors and uncertainties

Consider contribution of each simplex element to total variability

$$\varepsilon_{\mu} \lessapprox \sum_{j=1}^{n_e} \bar{\Omega}_j \left( \varepsilon_{\mathrm{AUQ}_j} + \varepsilon_{\mathrm{EUQ}_j} + \varepsilon_{\Delta \mathrm{x}_j} \right)$$

- Aleatory propagation/error estimation: Simplex SSC
- Numerical discretization error: Adjoints (space + time)
- Epistemic uncertainties: turbulence model perturbation



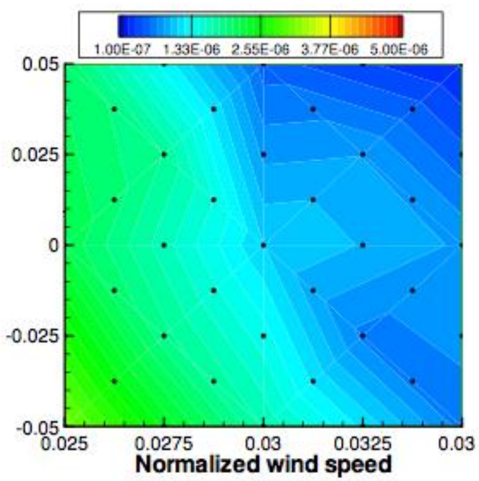




Figure : Stochastic

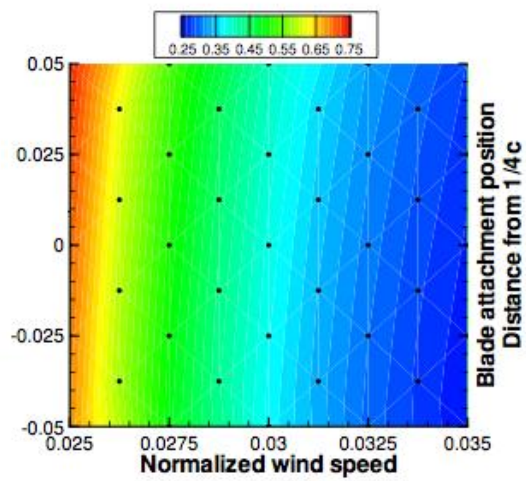



Figure : Spatial error

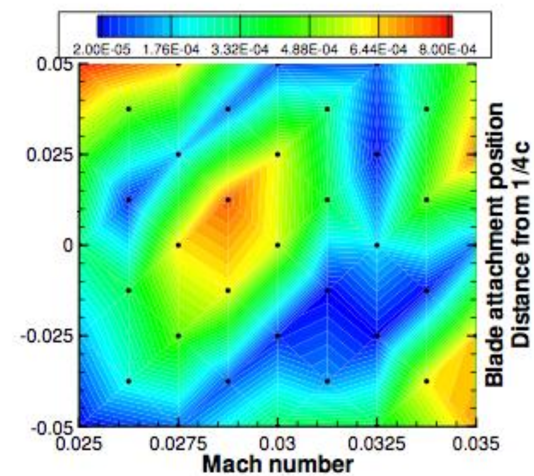
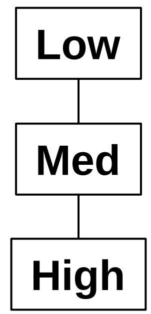



Figure : Temporal error

# Core-Enabled UQ: Multiple Model Forms

Same physics: (multiphysics, multi-scale provide additional dimensions in model "tree")

- A clear hierarchy of fidelity (low to high) → multifidelity UQ
  - Leverage concepts from provably-convergent multifidelity surrogate-based opt.




- An ensemble of models that could all be credible (lacking a clear preference structure)
  - model form uncertainty (inadequate data) → extend mixed aleatory-epistemic UQ capabilities to include categorical model forms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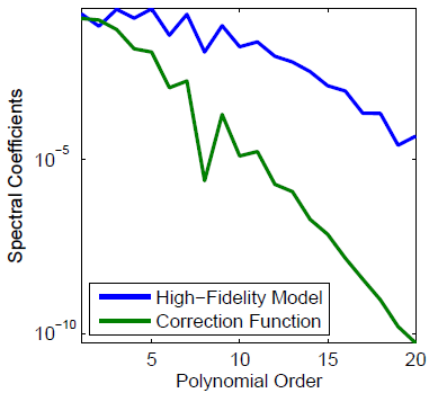
# Multifidelity UQ

*Multifidelity UQ through stochastic expansion of model discrepancy:*

- Extension of multifidelity opt methods that converge to local HF optimum based on local corrections
- Converge to global HF statistics based on global corrections ($0^{th}/1^{st}$ consistency @HF collocation pts)

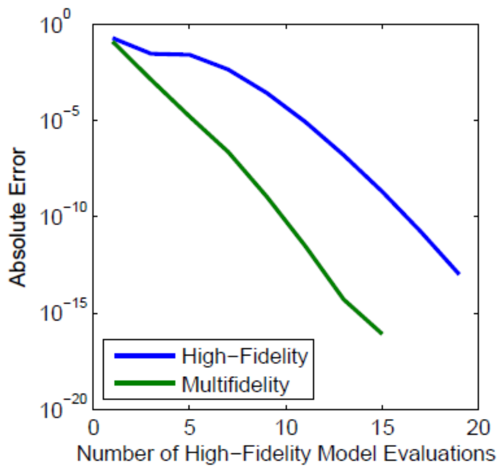
$$\hat{f_{hi}}(\xi) = \sum_{j=1}^{N_{lo}} f_{lo}(\xi_j) L_j(\xi) + \sum_{j=1}^{N_{hi}} \Delta f(\xi_j) L_j(\xi)$$

$$N_{lo} >> N_{hi}$$

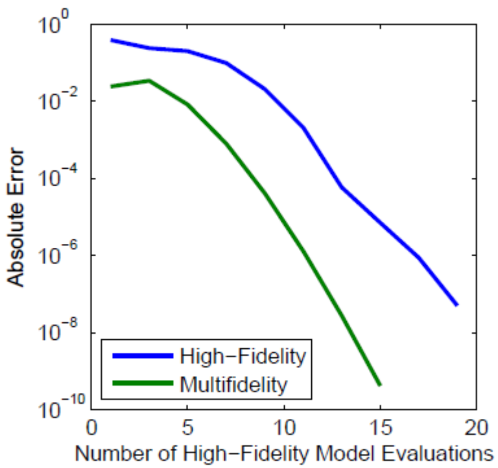



$$R_{\text{high}}(\xi) = e^{-0.05\xi^2} \cos 0.5\xi - 0.5 e^{-0.02(\xi-5)^2}$$

$$R_{\text{low}}(\xi) = e^{-0.05\xi^2} \cos 0.5\xi,$$



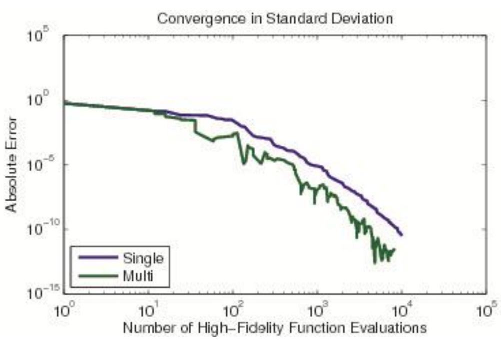

(a) Error in mean




(b) Error in standard deviation

*Adaptive algorithm balances LF/HF cost and targets regions where LF predictive capabilities break down:*

- Greedy selection of index sets for LF *or* model discrepancy based on ΔQOI/ΔCost



# Elliptic PDE with FEM

$$-\frac{d}{dx}\left[\kappa(x,\omega)\frac{du(x,\omega)}{dx}\right] = 1, \qquad x \in (0,1), \qquad u(0,\omega) = u(1,\omega) = 0$$

$$\kappa(x,\omega) = 0.1 + 0.03\sum_{k=1}^{10}\sqrt{\lambda_k}\phi_k(x)Y_k(\omega), \qquad Y_k \sim \text{Uniform}[-1,1]$$

$$C_{\kappa\kappa}(x,x') = \exp\left[-\left(\frac{x-x'}{0.2}\right)^2\right]$$

QoI is u(0.5, ω).

LF = coarse spatial grid with 50 states.

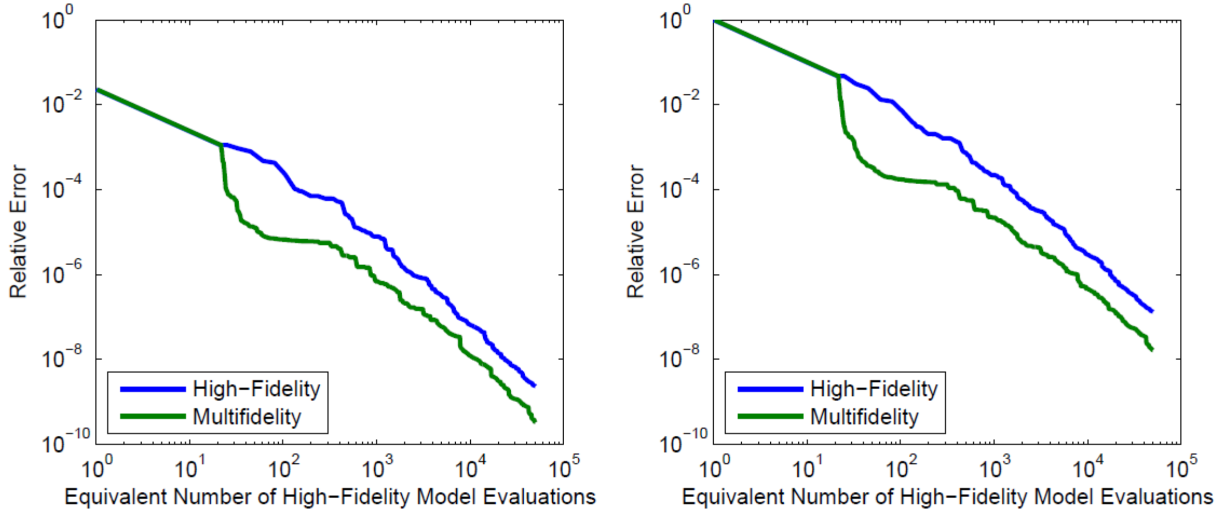
HF = fine spatial grid with 500 states.

$r_{work}$ = 40.

**Defined offset**

| | Relative Error in Mean | Relative Error in Std Deviation | High-Fidelity Evaluations | Low-Fidelity Evaluations |
|---|---|---|---|---|
| Single-Fidelity ($q=3$) | $5.3\times10^{-6}$ | $2.7\times10^{-4}$ | 1981 | – |
| Single-Fidelity ($q=4$) | $4.1\times10^{-7}$ | $2.3\times10^{-5}$ | 12,981 | – |
| Multifidelity ($q=4$, $r=1$) | $4.7\times10^{-7}$ | $2.6\times10^{-5}$ | 1981 | 12,981 |

**Adaptive**




(a) Error in mean

(b) Error in standard deviation

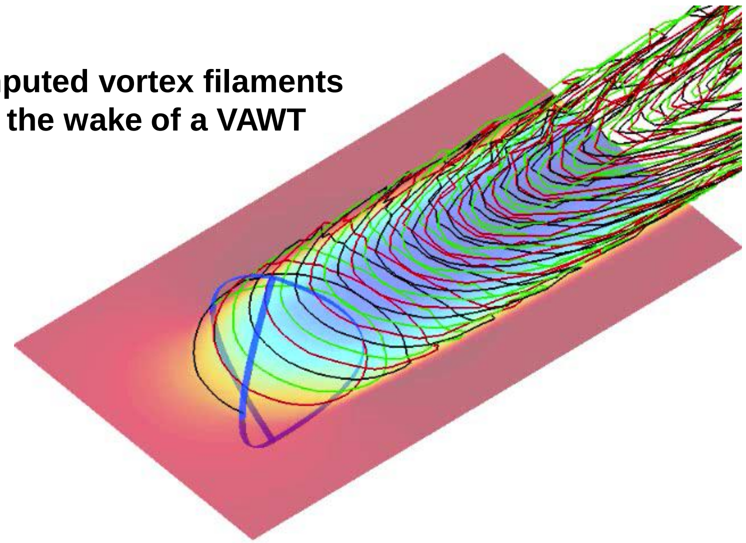
# Current Focus: VAWT Performance Modeling

**Vertical-axis Wind Turbine (VAWT)**

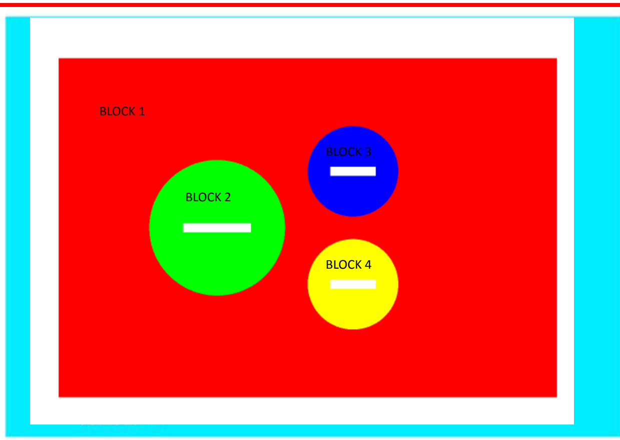

**Low fidelity**

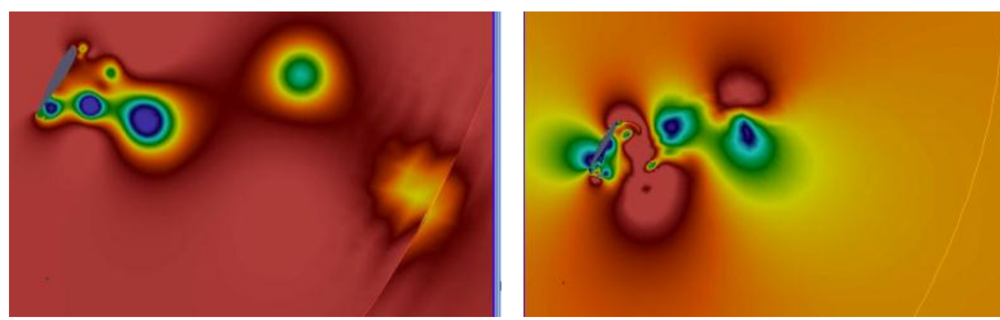
**CACTUS: Code for Axial and Crossflow TUrbine Simulation**

**Computed vortex filaments in the wake of a VAWT**



**High fidelity: DG formulation for LES**



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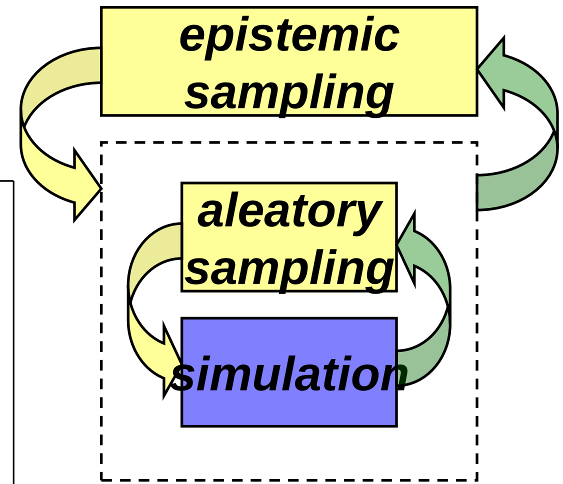
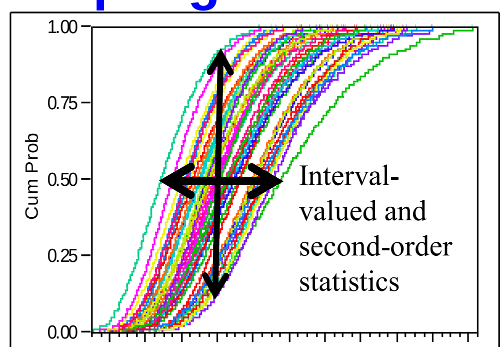
# Mixed Aleatory-Epistemic UQ:
## IVP, SOP, and DSTE based on Stochastic Expansions

Epistemic uncertainty (aka: subjective, reducible, lack of knowledge uncertainty): insufficient info to specify objective probability distributions




## Traditional approach: nested sampling

- Expensive sims → under-resolved sampling (especially @ outer loop)
- Under-prediction of credible outcomes




## Algorithmic approaches

- Interval-valued probability (IVP), *aka* probability bounds analysis (PBA)
- Dempster-Shafer theory of evidence (DSTE)
- Second-order probability (SOP), *aka* probability of frequency

Increasing epistemic structure (stronger assumptions)

## Address accuracy and efficiency

- Inner loop: stochastic exp. that are epistemic-aware (aleatory, combined)
- Outer loop:
  - IVP, DSTE: opt-based interval estimation, global (EGO) or local (NLP)
  - SOP: nested stochastic exp. (nested expectation is only post-processing in special cases)

$$\begin{array}{ll} \text{minimize} & M(s) \\ \text{subject to} & s_L \leq s \leq s_U \end{array}$$

$$\begin{array}{ll} \text{maximize} & M(s) \\ \text{subject to} & s_L \leq s \leq s_U \end{array}$$

# Mixed Aleatory-Epistemic UQ:
## IVP, SOP, and DSTE based on Stochastic Expansions

| Interv Est Approach | UQ Approach | Expansion Variables | Evaluations (Fn, Grad) | Area | $\beta$ |
|---|---|---|---|---|---|

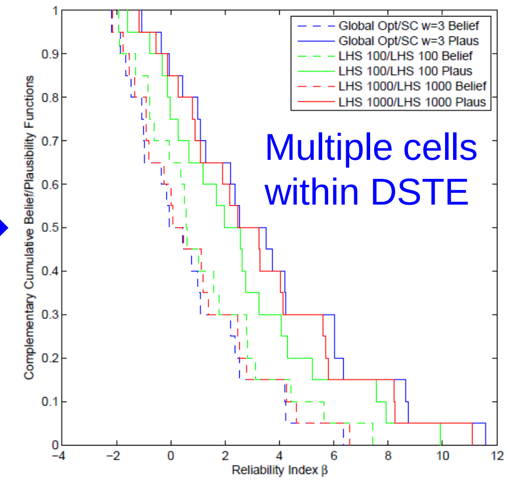
IVP SC SSG Aleatory: $\beta$ interval converged to 5-6 digits by 300-400 evals

| | | | | | |
|---|---|---|---|---|---|
| EGO | SC SSG w = 1 | Aleatory | (84/91, 0/0) | [75.0002, 374.999] | [-2.26264, 11.8623] |
| EGO | SC SSG w = 2 | Aleatory | (372/403, 0/0) | [75.0002, 374.999] | [-2.18735, 11.5900] |
| EGO | SC SSG w = 3 | Aleatory | (1260/1365, 0/0) | [75.0002, 374.999] | [-2.18732, 11.5900] |
| EGO | SC SSG w = 4 | Aleatory | (3564/3861, 0/0) | [75.0002, 374.999] | [-2.18732, 11.5900] |
| NPSOL | SC SSG w = 1 | Aleatory | (21/77, 21/77) | [75.0000, 375.000] | [-2.26264, 11.8623] |
| NPSOL | SC SSG w = 2 | Aleatory | (93/341, 93/341) | [75.0000, 375.000] | [-2.18735, 11.5901] |
| NPSOL | SC SSG w = 3 | Aleatory | (315/1155, 315/1155) | [75.0000, 375.000] | [-2.18732, 11.5900] |
| NPSOL | SC SSG w = 4 | Aleatory | (891/3267, 891/3267) | [75.0000, 375.000] | [-2.18732, 11.5900] |

IVP nested LHS sampling: converged to 2-3 digits by $10^8$ evals

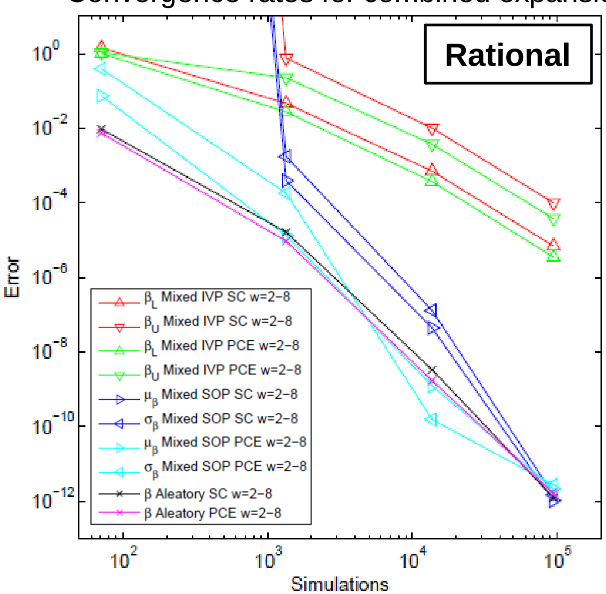
| | | | | | |
|---|---|---|---|---|---|
| LHS 100 | LHS 100 | N/A | $(10^4/10^4, 0/0)$ | [80.5075, 338.607] | [-2.14505, 8.64891] |
| LHS 1000 | LHS 1000 | N/A | $(10^6/10^6, 0/0)$ | [76.5939, 368.225] | [-2.19883, 11.2353] |
| LHS $10^4$ | LHS $10^4$ | N/A | $(10^8/10^8, 0/0)$ | [76.4755, 373.935] | [-2.16323, 11.5593] |

Fully converged area interval = [75., 375.], $\beta$ interval = [−2.18732, 11.5900]



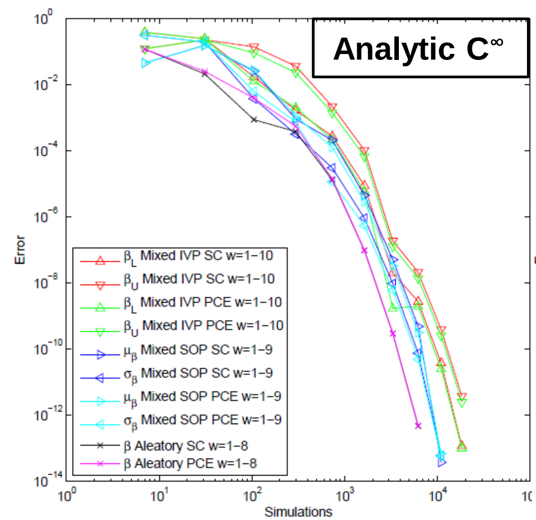

Multiple cells within DSTE

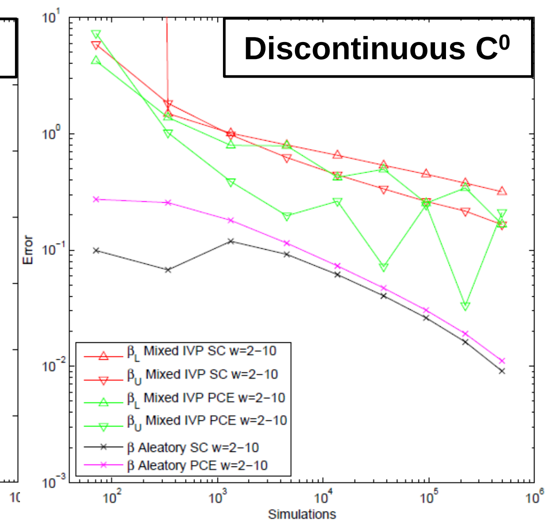
Convergence rates for combined expansions




$L^\infty$ metrics: IVP mixed, DSTE mixed

$L^2$ metrics: Aleatory, SOP mixed

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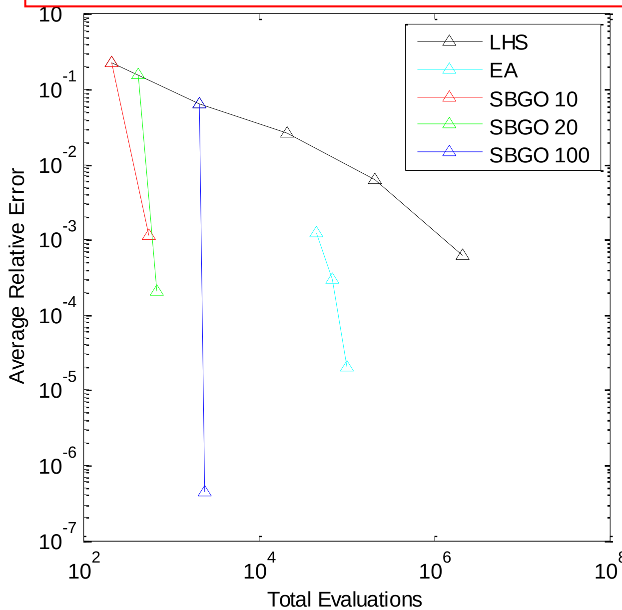
# Addition of Discrete Epistemic Model Form

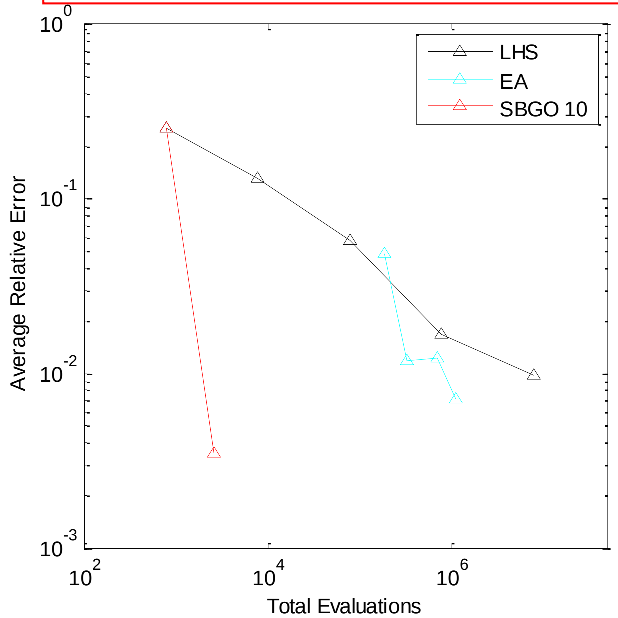
## MINLP interval estimation approaches

- Latin hypercube sampling (LHS)
- Evolutionary algorithm (EA)
- Surrogate-based global optimization (SBGO)

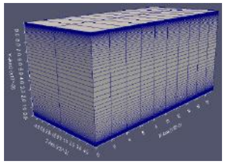
$$\text{Form 1}: \quad f_1 = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$
$$\text{Form 2}: \quad f_2 = 100(x_2 - x_1^2 + .2)^2 + (0.8 - x_1)^2$$

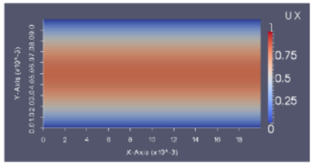



$$\text{Form 1}: \quad f_1 = 1 - \frac{4M}{bh^2Y} - \left(\frac{P}{bhY}\right)^2$$
$$\text{Form 2}: \quad f_2 = 1 - \frac{4P}{bh^2Y} - \left(\frac{P}{bhY}\right)^2$$
$$\text{Form 3}: \quad f_3 = 1 - \frac{4M}{bh^2Y} - \left(\frac{M}{bhY}\right)^2$$
$$\text{Form 4}: \quad f_4 = 1 - \frac{4M}{bh^2Y} - \left(\frac{P}{bhY}\right)^2 - \frac{4(P-M)}{bhY}$$




**Drekar RANS turbulence:** Spalart-Allmaras, k-ε






| Method | Outer Evals | Total Evals | $\mu_{ux}$ | $\mu_{pressure}$ |
|---|---|---|---|---|
| LHS | 10 | 250 | [0.727604, 2.78150] | [32.6109, 282.237] |
| SBGO | 17 | 425 | [0.622869, 4.44624] | [21.7321, 297.957] |

# Summary

## *Remarks on UQ*

- We are developing a broad suite of scalable and robust core UQ methods
  - adaptive refinement, adjoint enhancement, & sparsity detection on structured & unstructured grids
  - framework for balancing errors among deterministic and stochastic sources
- We are now building on this foundation with multifidelity and model form UQ

## *Remarks on wind turbine simulation*

- We have deployed UQ methodologies to current production wind simulation tools
  - Design codes from NREL and SNL: FAST, EOLO, CACTUS
  - LF tools have exhibited nonsmooth behavior, particularly when modeling turbulent in-flows and gust loading, motivating an increased emphasis on algorithmic robustness (local h-refinement)
- HF simulation tools (e.g., DG LES) are coming online
  - assess reliability in design limiting environments
  - anchor low fidelity results in the multifidelity setting

## *Impact and deployment*

- UQ tools deployed through DAKOTA (v5.3 releasing 1/31/13)
- NREL systems analysis tool is leveraging OpenMDAO and DAKOTA
- Sandia: DAKOTA is being deployed to Sandia-led efforts in EERE's Wind Power Program

## *Directions*

- Leadership-class CFD (towards exascale) for complex configurations & multiple turbines
  - leveraging low fidelity design codes in a rigorous manner
- Exploring partnership opportunities to strengthen ties of UQ/OUU R&D to wind energy